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A brief review given of temperature scales, their definition, and measurement. Methods of practical temperature measurement are described. Differences between internationally accepted temperature scales are presented graphically and in tabular form; comparisons are also given for several national temperature scales. Emphasis has been placed on temperatures below 0 C, but some information on temperatures above 0 C has also been included. Suggested methods of conversion between different temperature scales are described.

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A COMPILATION AND HISTORICAL REVIEW OF TEMPERATURE SCALE DIFFERENCES

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SPACE AGE science and technology have brought with them the need for more accurate and precise data on properties of materials. Data accurate to small fractions of a percent are no longer only of academic interest or unique. They are needed in sophisticated but practical engineering design. These data may come from new measurements or from compilation and evaluation of older measurements. For the current measurements to be sufficiently accurate and precise, the standards of measurement, upon which they are based, must be constantly re-examined. One such standard of measurement is the temperature scale. Recently the international practical standard of temperature measurement was redefined to satisfy modern technology.

There is not sufficient time for new measurements to satisfy all of the immediate demands for material property data. Therefore, the older data in the literature can be of significant value if properly compiled and evaluated. Recently, many compilation and evaluation groups have been formed to critically evaluate data and select best values. Researchers in such groups must intercompare data from the various sources in the literature. However, these intercomparisons are valid only if the measurements are based on the same standards of measurements. In the case of temperature, standards have progressively changed with improved experimental techniques and, in some instances, different standards have existed simultaneously in different countries. Thus, the critical evaluater of data must be concerned about the differences in temperature standards by which the data were determined. This paper is intended to help clarify the somewhat confusing situation regarding the development of temperature measurement and to compile data regarding the differences between the more common temperature scales.

An interesting historical account of the concept of temperature and its measurement is presented by R. B. Lindsay.[1] As described by him, temperatures were, at first, only thought of qualitatively and described as either hot or cold. Later came the desire to develop instruments that could quantitatively indicate relative degrees of hot or cold. The scales on these early instruments were arbitrarily defined in terms of the change in a given property of a given substance when placed in contact with bodies of different degrees of hot or cold. Usually the relationship between temperature and the given property observed was taken as linear. Thus, temperature was defined to be proportional to the position of the meniscus of a column of fluid (water in glass, mercury in glass thermometer, etc.), or the resistance of a given wire (resistance thermometer), or the pressure of a gas in a confined volume (gas thermometer). But each of these scales differed from the others at most points except at the defining points; these differences existed, since temperature scales, so defined, are dependent upon the substance and property on which they are based.

To overcome these difficulties, William Thomson (Lord Kelvin) in 1854 proposed a thermodynamic temperature scale based upon the Carnot cycle which is independent of the working substance. This thermodynamic scale is recognized today as the fundamental temperature scale to which all other scales should be referable. The ratio of two temperatures on the thermodynamic scale is equal to the ratio of the heat absorbed and rejected, respectively, by a Carnot engine operating between two reservoirs at these temperatures.[2,3,4] It is important to note that the thermodynamic scale is derived from the laws of thermodynamics and, therefore, thermodynamic relationships are strictly valid only for temperatures defined by this scale. From about the time of Kelvin's proposal, most recognized practical temperature scales have been defined to be consistent with the thermodynamic temperature scale within the accuracy of the calibration techniques.

In the early part of this century the only internationally accepted practical scale was represented by a mercury-in-glass thermometer maintained by the Bureau International des Poids et Mesures (BIPM), Paris. The thermometer was calibrated by Chappuis by comparison with the constant volume hydrogen gas thermometer. Because of the limited range (0–100 C) and

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reproducibility (±0·01 C) of this internationally accepted scale, many other scales came into existence. In order to avoid further proliferation of temperature scales, in 1911 the directors of the National Laboratories of Germany, Great Britain, and the United States agreed to strive for an internationally acceptable standard. This effort resulted in the adoption of the International Temperature Scale[5] (ITS) in 1927 by the Seventh General Conference on Weights and Measures. In 1937 the Advisory Committee on Thermometry, a working group under the International Committee on Weights and Measures, was established to investigate and recommend improvements and extension of the ITS. Because of the difference between the thermodynamic and ITS scales (detected shortly after the adoption of the ITS) and because the ITS did not extend below the boiling point of oxygen, several laboratories established and maintained their own temperature scales. Also, as time progressed individual laboratories made changes in their temperature scales. We, thus, find today published data on properties of materials as a function of temperature based on temperature scales differing from each other by detectable amounts. These differences are generally small and sometimes are not even significant; but in some instances, especially when data are highly accurate, comparisons of data should be made on a common temperature scale and in thermodynamics where first and second derivatives with respect to temperature are applied to calculate derived properties, it is important to be as close as possible to the thermodynamic temperature scale. Brickwedde[6] has cited instances wherein apparent irregularities in the low temperature variation of physical properties of materials were explained later by errors in the representation of the thermodynamic temperature scale.

The purpose of this work is to compile information on the differences between the more common temperature scales which have been in use since the latter part of the 19th century. This compilation will be an aid in converting reported temperatures to the thermodynamic temperature scale or from one scale to another, thus, eliminating temperature inconsistencies, insofar as possible, when intercomparing or representing property data. It is expected that the primary users of these comparisons will be individuals frequently preparing correlations of data, for example, the members of the Cryogenic Data Center, National Bureau of Standards, Boulder, Colo. It is hoped that those who encounter these problems less frequently may also find this report helpful. This investigation, performed primarily for temperatures below the ice point, also includes some data on temperature scale differences at higher temperatures.

First, I will review the definition and realization of the thermodynamic temperature scale. Next, the origin and basis of practical temperature scales will be presented. Finally, I shall present intercomparisons of thermodynamic and practical temperature scales along with conversions to the present best estimate of the thermodynamic scale.

Thermodynamic temperature scales

As pointed out in the preceding section, the Carnot cycle serves only to define the ratio of two temperatures on the thermodynamic scale. To define a unit of temperature, one must assign a number to the temperature difference between two fixed points. Kelvin wisely proposed that at first the temperature interval from the ice point to the steam point be defined as 100 degrees, consistent with earlier centigrade scales. This scale was called the Kelvin thermodynamic scale. In 1954 the definition of the Kelvin scale was very slightly changed so that the interval from absolute zero to the triple point of water was exactly 273·16 degrees. Kelvin indicated in his original proposal that ultimately this should be done. Thus, before 1954, all absolute Kelvin scale temperatures were a matter for experimental determination; but, since 1954, the triple point of water has been defined as exactly 273·16 K. The value of 273·16 K was chosen as the most probable value for the triple point based upon a correlation of all the measurements made on the pre-1954 scale. The interval from the ice point to the steam point is no longer exactly 100 degrees, but, until new measurements show otherwise, is still 100 degrees within experimental uncertainty (approximately 5 mK). Practically speaking, pre-1954 and post-1954 Kelvin scales are the same except that a given pre-1954 scale may have been based on an ice point different from 273·15 K. A pre-1954 Kelvin scale temperature, T^1, based on an ice point temperature of T_0^1 may be converted to a post-1954 Kelvin scale temperature, T, using (1)

$$T = T^1 \frac{273{\cdot}15}{T_0^1} \qquad \ldots (1)$$

where 273·15 K is the temperature of the ice point on post-1954 Kelvin scale (subject to experimental error).

Often temperatures were reported on scales other than the Kelvin scale, for example, Celsius, Fahrenheit, and Rankine. The Celsius temperature scale is defined to have the same size degree as the Kelvin scale. The triple point of water is now defined as 0·01 C; formerly, the ice point was taken as 0 C. The Rankine thermodynamic scale (an absolute scale) is defined such that the interval from the ice point to the boiling point of water is 180 degrees. The Fahrenheit scale is defined to have the same size degree as the Rankine scale; but the ice point is assigned the value of 32 F and, thus, the steam point is 212 F. Table 1 contains the conversion formulas for these scales.

The definition of the thermodynamic temperature scale based upon the idealized Carnot cycle does not constitute a practical operational definition. It remains to relate this definition to measurable physical variables.

In the following sections, various methods of realizing the thermodynamic scale will be reviewed. Sources of error will also be considered in the section on gas thermometry to establish the basis for temperature scale differences.

TABLE 1. CONVERSION FORMULAS FOR THERMODYNAMIC TEMPERATURE SCALES (ICE POINT = 273·15 K)

Temperature in degrees =	*Kelvin*	*Celsius*	*Rankine*	*Fahrenheit*
Kelvin	T	$T + 273{\cdot}15$	$T/1{\cdot}8$	$(T + 459{\cdot}67)/1{\cdot}8$
Celsius	$T - 273{\cdot}15$	T	$(T - 491{\cdot}67)/1{\cdot}8$	$(T - 32)/1{\cdot}8$
Rankine	$1{\cdot}8T$	$(T + 273{\cdot}15)1{\cdot}8$	T	$T + 459{\cdot}67$
Fahrenheit	$1{\cdot}8T - 459{\cdot}67$	$1{\cdot}8T + 32$	$T - 459{\cdot}67$	T

T = temperature of scale across top of table

Gas thermometry. Herzfeld[4] shows that the thermodynamic temperature scale, as defined by Kelvin, is the same as a temperature scale based on an ideal gas. An ideal gas is defined by two properties: 1. Boyle's law—the product of pressure times volume of an ideal gas is a function of temperature only and 2. Joule's law—the temperature of an ideal gas does not change upon expansion into a vacuum, that is, without doing work.

Since a real gas, at the limit of zero pressure, behaves as an ideal gas, one can obtain the thermodynamic Kelvin scale from

$$T = T_0 \lim_{P \to 0} \frac{(PV)_T}{(PV)_{T_0}} \qquad \ldots (2)$$

where $(PV)_T$ is the pressure–volume product for a real gas evaluated at temperature, T, and $(PV)_{T_0}$ is evaluated at the triple point of water. The form of (2) is based on the post-1954 Kelvin scale definition wherein only the triple point of water is a defined fixed point. The equivalent form on the pre-1954 scale is

$$T = \lim_{P \to 0} \frac{100(PV)_T}{(PV)_{100} - (PV)_0} \qquad \ldots (3)$$

where the subscripts 100 and 0 indicate the steam point and ice point, respectively. In the older literature this was called the Avogadro temperature scale. The limiting value of the ratios in (2) and (3) can be obtained either by extrapolating a series of values to zero pressure or by making corrections for the non-ideality of the real gas.

One significant source of error in the determination of thermodynamic temperatures arises from the need to correct for the non-ideality of the gas. Temperature corrections for the non-ideality of the nitrogen, hydrogen, and helium gas scales have been tabulated by Keyes[7] and van Dijk,[8] and are illustrated in Figures 1, 2, and 3. These figures show that gas thermometer temperatures corrected for non-ideality from various laboratories may differ from each other by as much as 0·04 K. These differences are caused by the differences in the values of the second virial coefficients or Joule–Thomson data considered best by the various workers.

PV isotherm thermometry, a type of gas thermometry, involves the measurement of PV values for a given gas along an isotherm. These PV values are extrapolated to zero pressure yielding a value for RT. Dividing by the gas constant, R, yields the temperature, T, of that isotherm. This method eliminates the necessity of making corrections for the non-ideality of the gas.

Another source of significant error in the determination of thermodynamic temperature is the adjustment made to correct for the non-ideality of the apparatus. This correction includes dead space volume, gravitational effects, thermomolecular pressure effects, adsorption effects, and expansion of the thermometer bulb. Since this correction is dependent upon the specific details of each apparatus, it is not tabulated here; however, temperature differences caused by uncertainty in this correction are comparable to those caused by the correction for the non-ideality of the gas. For detailed information regarding the mechanics and calculations of gas thermometry, the reader is referrred to papers by Beattie,[9] Moser,[10,11] Moser, Otto, and Thomas,[12,13] Moessen, Aston, and Ascah,[14] Barber,[15] Kirenkov,[16]

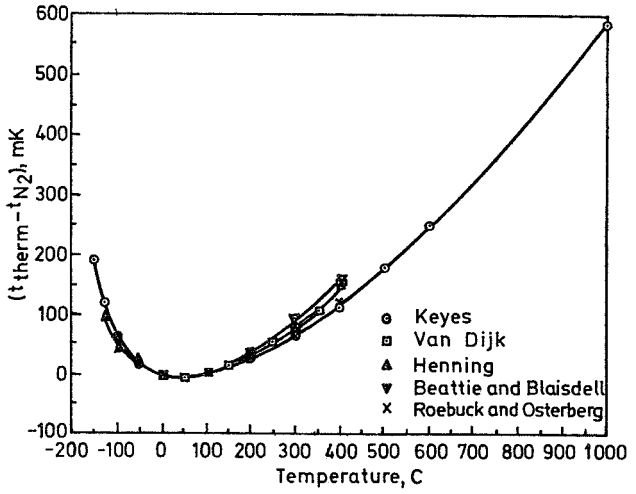

Figure 1. Gas imperfection corrections for constant volume nitrogen gas thermometer

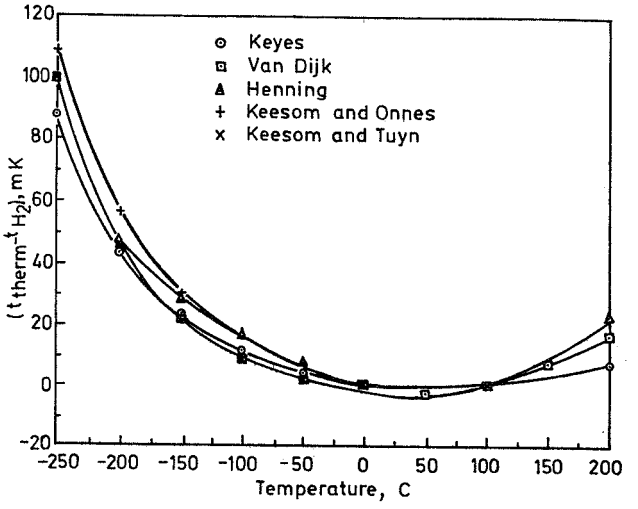

Figure 2. Gas imperfection corrections for constant volume hydrogen gas thermometer

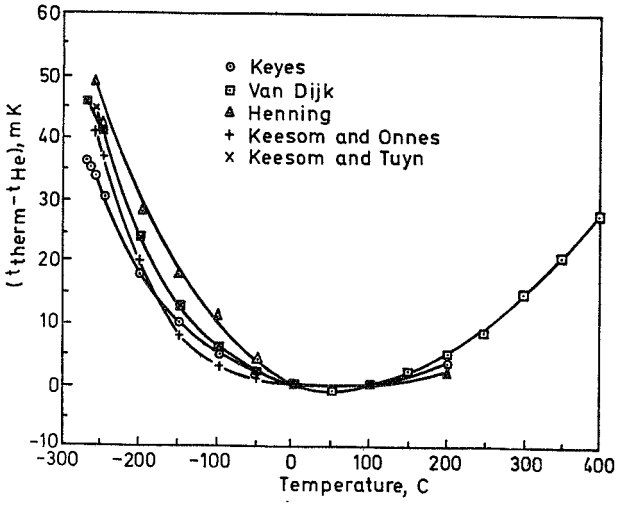

Figure 3. Gas imperfection corrections for constant volume helium gas thermometer

Guildner,[17] Onnes and Braak,[18] Kinoshita and Oishi,[19] and Borovick-Romanov, Strelkov, Orlova, and Astrov.[20]

Vapour pressure thermometry. Ziegler, Mullins, and Kirk[21] and van Dijk[22] have compiled equations relating the real gas PVT properties, specific heats, and latent heats of vapourization from which one can obtain the statistical thermodynamic vapour pressure temperature relation. This relation may be used for the establishment of a vapour pressure scale resulting in another way to realize the thermodynamic temperature scale. This should not be confused with the more common experimental procedure of calibrating a vapour pressure thermometer with a gas thermometer.

Acoustical thermometry. Another method of determining thermodynamic temperatures is by measuring the speed of sound in a real gas and extrapolating to zero pressure to obtain the ideal gas value. Zemansky[2] shows that for an ideal gas the speed of sound of sufficient wavelength is related to temperature according to (4).

$$W^2 = \frac{\gamma RT}{M} \qquad \ldots (4)$$

where W = speed of sound, γ = ratio of specific heats = C_p/C_v, T = thermodynamic temperature, M = molecular weight, and R = the universal gas constant. For further information regarding this type of thermometry, the reader is referred to Cataland, Edlow, and Plumb,[23] Hedrick and Pardue,[24] and Plumb and Cataland.[25,26]

Magnetic thermometry. At very low or high temperatures, below about 1 K and above about 1 000 K, gas thermometry becomes impractical. At low temperatures, one can realize the thermodynamic scale from the magnetic properties of paramagnetic solids. For many substances the magnetic susceptibility, χ, depends on temperature according to (5).

$$\chi = \frac{C}{T^* + \Delta} + \alpha \qquad \ldots (5)$$

where C is the Curie constant, Δ is the Weiss constant, and α is also temperature independent.

Because the constants in (5) are difficult to determine accurately, the temperature scale, T^*, so defined, is not exactly the thermodynamic scale. However, by measuring the relation between heat energy added, Q, and T^*, along with the relation between entropy, S, and T^*, one can compute the thermodynamic temperature from the second law of thermodynamics, $dQ = TdS$, from (6).

$$T = \frac{dQ}{dS}\left(\frac{\partial Q/\partial T^*}{\partial S/\partial T^*}\right)_{H=0} \qquad \ldots (6)$$

where $H = 0$ denotes zero magnetic field. The complicated details of this procedure, not necessary here, have been described by de Klerk and Steenland[27] and Hudson.[28]

Optical thermometry. At high temperatures, above about 1 000 K, equation 7, based on the Planck radiation law, is used to determine temperatures on the thermodynamic scale. This range of temperature is not considered in detail in this paper. The interested reader is referred to *Temperature, Its Measurement and Control in Science and Industry*, Vols. 1, 2, and 3.*

$$\frac{L_\lambda(T)}{L_\lambda(T_{Au})} = \frac{\exp\left[\frac{c_2}{\lambda T_{Au}}\right] - 1}{\exp\left[\frac{c_2}{\lambda T}\right] - 1} \qquad \ldots (7)$$

where $L_\lambda(T)$ and $L_\lambda(T_{Au})$ = spectral concentrations at temperatures T and the freezing point of gold, T_{Au}, of the radiance of a black body at the wavelength λ, c_2 = 0·014388 metre Kelvin.

Practical temperature scales

The previous section described methods to determine temperatures on the thermodynamic temperature scale. But temperature measurement with these methods generally requires excellent laboratory facilities and techniques. Also, the computations that must be performed for accurate results are extensive and costly. Only a few laboratories in the world have the capabilities to make such measurements. Thus, it is recognized that practical scales, which approximate the thermodynamic scale but are easier to use, are required.

International practical temperature scales

In 1927 the International Temperature Scale (ITS) was adopted for practical thermometry. The fixed points, interpolation instruments, and interpolation formulas for the ITS are presented in Tables 2 and 3. The ITS was defined to approximate the thermodynamic scale as closely as experimentally possible. The ITS does not replace the thermodynamic scale. In fact, it was intended that whenever differences between the ITS and thermodynamic scale could be conclusively determined, the ITS would be changed to eliminate these differences. The first such change became reality in 1948. This revised scale is known as the International Temperature Scale (1948). The revision involved the following important changes: (a) the silver point was changed from 960·5 to 960·8 C, (b) the Planck radiation formula replaced the Wien formula for temperatures above the gold point, and (c) the platinum thermometer range was restricted to temperatures above the oxygen boiling point. These changes affected the numerical values of the scale only above 630 C. At this time the name Celsius was adopted in place of centigrade or centesimal.

In 1960 a text revision of the 1948 scale[30] was adopted by the Eleventh General Conference of Weights and Measures. This revision did not appreciably affect the numerical values of the scale. The principal changes were (1) to replace the ice point, as a fixed reference point, with the triple point of water (0·01 C exactly), (2) the recommendation to use the zinc point as an alternative to the sulphur point, and (3) to change the name to International Practical Temperature Scale (IPTS). The ITS was a Celsius (centigrade) scale, and no mention is made in the original text of a 'practical Kelvin scale.' However, the conversion from C to K was

*Published by Reinhold Publishing Corporation, New York (1941, 1955, and 1962).

made in practice with the following equation

$$T(\text{Int})\ \text{K} = t(\text{Int})\ \text{C} + T_0 \qquad \ldots (8)$$

where T_0 is the temperature of the ice point on the Kelvin thermodynamic scale. The value of T_0 was generally taken to be either 273·15 K or 273·16 K. The (Int) has usually been omitted from T(Int)K, implying correspondence with the Kelvin thermodynamic temperature scale. The omission of this subscript from the literature makes it difficult for a reader to know if the reported temperatures are thermodynamic or practical temperatures.

The 1960 revision of the 1948 scale[30] specifically mentions the International Practical Kelvin Temperature Scale and relates it to the IPTS via (8). The value of T_0 was defined as 273·15. This was done so that the International Practical Kelvin Temperature Scale had the same value (273·16 K) as the Kelvin thermodynamic scale at the triple point of water. The IPTS and Celsius thermodynamic scales were also both defined to have the value of 0·01 C at the triple point of water. The fixed points and interpolation methods of the international scales are summarized in Tables 2 and 3. Complete descriptions of these scales are given by Stimson,[29,30] Burgess,[5] and Corruccini.[31]

During the past few years members of the Advisory Committee on Thermometry (Comité Consultatif de Thermométrie), CCT, have been actively compiling and correlating data on the deviations of the IPTS from the thermodynamic temperature scale. This effort resulted in the adoption (October 1968) of a new international practical temperature scale by the International Committee of Weights and Measures. The new scale, designated as IPTS–68, is defined in terms of a series of fixed points, a resistance versus temperature reference function, and interpolation formulas for specified instruments used to realize the scale. It differs from the previous scale, IPTS–48, in the following ways.

1. The IPTS–68 extends downward to 13·8 K, whereas the IPTS–48 terminated at 90·18 K.

2. New values have been assigned to most of the defining fixed points and to the second radiation constant, c_2, (see Table 3) in order to more closely approximate the thermodynamic temperature scale.

The IPTS–68 is the present best estimate of the thermodynamic scale and will be treated as such in comparisons in later sections of this paper. The fixed points, interpolation formulas, and interpolation instruments of the IPTS–68 are presented in Table 4.

Further details on the history of practical temperature measurements are given by Hall.[33–36]

TABLE 2. PRIMARY REFERENCE POINTS FOR THE INTERNATIONAL TEMPERATURE SCALES

Description (all at 1 atm) except triple point of water	*C(Int 1927)*	*C(Int 1948)*	*C(Int 1948) 1960 text rev.*
Oxygen point: equilibrium between liquid and gaseous oxygen	−182·970	−182·970	−182·97
Ice point: equilibrium between ice and air saturated liquid water	0·000	0	
Triple point of water: equilibrium between ice, liquid water and gaseous water			0·01
Steam point: equilibrium between liquid and gaseous water	100·000	100	100
Sulphur point: equilibrium between liquid and gaseous sulphur	444·60	444·600	444·6
Zinc point: equilibrium between solid and liquid zinc			419·505*
Silver point: equilibrium between solid and liquid silver	960·5	960·8	960·8
Gold point: equilibrium between solid and liquid gold	1 063	1 063·0	1 063

* Recommended as a substitute for the sulphur point

TABLE 3. OFFICIAL INTERPOLATION PROCEDURES FOR THE 1927 AND 1948 INTERNATIONAL TEMPERATURE SCALES (t in C; T in K; T_0 IS TEMPERATURE OF ICE POINT)

1927 Scale	*1948 Scale*
−190 to 0 C platinum resistance thermometer $R_t = R_0[1 + At + Bt^2 + C(t-100)t^3]$ calibrate at ice, steam, sulphur, and oxygen points	−182·97 to 0 C platinum resistance thermometer $R_t = R_0[1 + At + Bt^2 + C(t-100)t^3]$ calibrate at ice, steam, sulphur, and oxygen points
0 to 660 C $R_t = R_0(1 + AT + Bt^2)$ $R_{100}/R_0 \geq 1{\cdot}390\ R_{444.6}/R_0 \geq 2{\cdot}645$ calibrate at ice, steam, and sulphur points	0 to 630·5 C $R_t = R_0(1 + At + Bt^2)$ $R_{100}/R_0 \geq 1{\cdot}3910$ calibrate at ice, steam, and sulphur points
660 to 1 063 C Pt–Pt 10% Rh thermocouple $e = a + bt + ct^2$ calibrate at Sb, Ag, and Au points	630·5 to 1 063 C Pt–Pt 10% Rh thermocouple $E = a + bt + ct^2$ calibrate at Sb, Ag, and Au points
above 1 063 C monochromatic optical pyrometer $\log \frac{J_t}{J_{Au}} = \frac{c_2}{\lambda}\left[\frac{1}{1336} - \frac{1}{t+T_0}\right]$ with $c_2 = 1{\cdot}432$ cm deg and $T_0 = 273$	above 1 063 C monochromatic optical pyrometer $\frac{J_t}{J_{Au}} = \frac{\exp\frac{c_2}{\lambda(t_{Au}+T_0)} - 1}{\exp\frac{c_2}{\lambda(t+T_0)} - 1}$ $c_2 = 1{\cdot}438$ cm deg T_0 = temperature of the ice point

National practical temperature scales between 10 and 90 K

Before the recently adopted IPTS–68, no international practical temperature scale existed in the range 5 to 90 K. However, significant research had been conducted in this range even before the beginning of this century. The accelerated pace of this research, in recent years especially, created the need for standardization of practical temperature measurement at these temperatures. Since no international scale had been established, several national laboratories established practical scales for use within each country. Each of these scales is based on gas thermometry and maintained through platinum resistance thermometers. These scales differ from each other due to (a) selection of different temperatures assigned to the fixed points, (b) application of different values for the corrections due to the non-ideality of the

TABLE 4A. FIXED POINTS OF IPTS–68

	Equilibrium Hydrogen	*Neon*	*Oxygen*	*Water*
Triple point	13·81K		54·361K	273.16K
Boiling point	20·28K	27·102K	90·188K	373·15K
Liquid–vapour equilibrium at a pressure of 33, 330·6 N/m²	17·042K			
	Zinc		*Silver*	*Gold*
Freezing point	692·73K		1 235·08K	1 337·58K

TABLE 4B. THE REFERENCE FUNCTION $W_{CCT-68}(T_{68})$ FOR PLATINUM RESISTANCE THERMOMETERS FOR THE RANGE FROM 13·81 TO 273·15 K

$$T_{68} = \left\{A_0 + \sum_{i=1}^{20} A_i [\ln W_{CCT-68}(T_{68})]^i\right\} K$$

where $W_{CCT-68}(T_{68}) = R_{CCT}(T_{68})/R_{CCT-68}$ (273·15 K)

i	A_i	i	A_i
0	0·273 15 × 10³	11	0·767 976 358 170 845 8 × 10
2	0·250 846 209 678 803 3 × 10³	12	0·213 689 459 382 850 0 × 10
2	0·135 099 869 964 999 7 × 10³	13	0·459 843 348 928 069 3
3	0·527 856 759 008 517 2 × 10²	14	0·763 614 629 231 648 0 × 10⁻¹
4	0·276 768 548 854 105 2 × 10²	15	0·969 328 620 373 121 3 × 10⁻²
5	0·391 053 205 376 683 7 × 10²	16	0·923 069 154 007 007 5 × 10⁻³
6	0·655 613 230 578 069 3 × 10²	17	0·638 116 590 952 653 8 × 10⁻⁴
7	0·808 035 868 559 866 7 × 10²	18	0·302 293 237 874 619 2 × 10⁻⁵
8	0·705 242 118 234 052 0 × 10²	19	0·877 551 391 303 760 2 × 10⁻⁷
9	0·447 847 589 638 965 7 × 10²	20	0·117 702 613 125 477 4 × 10⁻⁸
10	0·212 525 653 556 057 8 × 10²		

TABLE 4C. INTERPOLATION PROCEDURES OF IPTS-68

Platinum resistance thermometer (13·81 to 273·15 K)
$W(T_{68}) = W_{CCT-68}(T_{68}) + \Delta W(T_{68}) = R(T_{68})/R(273{\cdot}15\ K)$

13·81 to 20·28 K

$\Delta W(T_{68}) = A_1 + B_1 T_{68}^3 + C_1 T_{68} + D_1 T_{68}^5$ constants are determined from measured values of $\Delta W(T_{68})$ at 13·81 K, 17·042 K, and 20·28 K; and by the derivative at 20·28 K of the deviation equation for the range 20·28 to 54·361 K.

20·28 to 54·361 K

$\Delta W(T_{68}) = A_2 + B_2 T_{68} + C_2 T_{68}^2 + D_2 T_{68}^3$ constants are determined from measured values of $\Delta W(T_{68})$ at 20·28 K, 27·102 K, and 54·361 K; and by the derivative at 54·361 K of the deviation equation for the range 54·361 to 90·188 K

54·361 to 90·188 K

$\Delta W(T_{68}) = A_3 + B_3 t_{68} + C_3 T_{68}^2$ constants are determined from measured values of $\Delta W(T_{68})$ at 54·361 and 90·188; and by the derivative at 90·188 K of the deviation equation for the range 90·188 to 273·15 K

90·188 to 273·15 K

$\Delta W(T_{68}) = A_4 t_{68} + C_4 t_{68}^3$ $(T_{68} - 100\ C)$; $t_{68} = T_{68} - 273{\cdot}15$ K constants are determined from measured values of $\Delta W T_{68})$ at 90·188 and 373·15 K

Platinum resistance thermometer (0 C to 630·74 C)
From 0 C to 630·74 C t_{68} is defined by

$$t_{68} = t' + 0{\cdot}045 \left(\frac{t'}{100\ C}\right)\left(\frac{t'}{100\ C} - 1\right) \cdot \left(\frac{t'}{419{\cdot}58\ C} - 1\right)\left(\frac{t'}{630{\cdot}74\ C} - 1\right) C$$

where t' is defined by the equation

$$t' = \frac{1}{a}[W(t') - 1] + \delta\left(\frac{t'}{100\ C}\right)\left(\frac{t'}{100\ C} - 1\right)$$

where

$$W(t') = \frac{R(t')}{R(0\ C)}$$

The constants $R(0\ C)$, a and δ are determined by measurement of the resistance at the triple point of water, the boiling point of water (or the freezing point of tin), and the freezing point of zinc.

Platinum versus rhodium–platinum thermocouple (630·74 C to 1 064·43 C)

$E(t_{68}) = a + bt_{68} + ct_{68}^2$ constants are determined by measuring $E(t_{68})$ at 630·74, 961·93, and 1 064·43 C with one junction at 0 C.

Radiation low range (above 1 337·58 K)

$$\frac{L\lambda(T_{68})}{L\lambda(T_{68}(Au))} = \frac{\exp\left[\frac{c_2}{\lambda T_{68}(Au)}\right] - 1}{\exp\left[\frac{c_2}{\lambda T_{68}}\right] - 1}$$

where $L\lambda(T_{68})$ and $L\lambda(T_{68}(Au))$ are the spectral concentrations at temperature T_{68} and at the freezing point of gold, $T_{68}(Au)$, of the radiance of a blackbody at the wavelength λ; $c_2 = 0{\cdot}014388$ metre kelvin.

gas and the apparatus, and (c) experimental errors. The recently adopted IPTS–68 is based on these national practical temperature scales in the range 13 to 90 K.

NBS scale. In 1939 Hoge and Brickwedde[37,38] calibrated six platinum and one 90% platinum +10% rhodium resistance thermometers with a helium gas thermometer. Gas imperfection corrections were applied as obtained from the mean values of second virial coefficients reported by Keesom[39] and Henning.[40] The boiling point of oxygen was taken to be 90·19 K while the triple point and boiling point of normal hydrogen were found to be 13·96 K and 20·39 K. This scale is known as the NBS–39 scale. In 1955 NBS revised the NBS–39 scale by subtracting 0·01 K throughout the range. Thus, on the NBS–55 scale the boiling point of oxygen is 90·18 K. This change was made because the oxygen boiling point, as obtained in 1939, appeared to be too high by about 0·01 K.

NPL scale. In 1962 Barber[41,42,44] established a low temperature scale at the National Physical Laboratory in England. This scale, extending from 10 to 90 K, is maintained by eight platinum resistance thermometer which were calibrated using a helium gas thermometer. Gas imperfection corrections were made using second virial coefficient data from Keesom.[43] The temperature of the oxygen boiling point was taken to be 90·18 K.

PSU scale. In 1953 Aston and Moessen[45] reported on the establishment of the Pennsylvania State University scale from 10 to 90 K. Seven platinum thermometers were calibrated using a helium gas thermometer. Gas imperfection corrections were based on the second virial coefficient data of Keyes.[7] The reference temperature used to establish this scale was 273·16 K for the ice point. The oxygen boiling point was then measured as 90·154 K.

PRMI scale. In 1954 Borovick-Romanov et al[20] estimated a Russian national scale for temperatures from 10 to 90 K. This scale is maintained by four platinum resistance thermometers calibrated with a constant volume helium gas thermometer. Gas imperfection corrections were based upon the second virial coefficient data of Keyes.[7] The oxygen boiling point was assumed to be 90·19 K. The boiling temperature of normal hydrogen was determined to be 20·391 K. In 1965 Orlova et al[46] of PRMI reported a new determination of the oxygen boiling point as 90·165 K. Gas imperfection corrections were made this time using the second virial coefficient data of Keesom.[43] Measurements based on the PRMI scale after 1965 may be based on this oxygen boiling point.

Helium vapour pressure scales

The vapour pressure of helium has long been used to determine temperatures below 5·2 K. The first helium vapour pressure scale was established in 1924. As the vapour pressure–temperature relation of liquid helium became better known through the years, improved temperature scales resulted. The most recent of the He^4 scales is referred to as the 1958 He^4 scale and was recommended for general use as the international stan-

dard of temperature by the 1958 General Conference of Weights and Measures. This scale extends from 1 to 5·2 K and was obtained by correlating measurements with magnetic thermometry, gas thermometry, PV isotherms, and thermodynamic vapour pressure thermometry as described in the section entitled 'Thermodynamic temperature scales'. Clement[47] examined the 1958 He^4 scale for roughness; his conclusion was that the scale is accurate to within 1 or 2 mK and the roughness is less than 0·1 mK.

Below 2·2 K, liquid He^4 is in a superfluid state which causes difficulties in accurate vapour pressure thermometry. To circumvent this difficulty and to extend practical thermometry to lower temperatures, the He^3 vapour pressure scale was established by Sydoriak et al.[48] The International Committee on Weights and Measures recommended in 1962 the general usage of this He^3 scale; and in the region of overlap with the He^4 scale, it is recommended that both scales can be used concurrently. The 1962 He^3 scale extends from 0·2 to 3·2 K.

NBS acoustical scale

In 1965 and 1966 Plumb and Cataland[26,49,50] reported on the establishment of a provisional NBS scale based on their acoustical thermometry in the range of 2 to 20 K. This scale, referred to as NBS P 2–20 (1965), is maintained by six germanium resistance thermometers which were calibrated against the acoustical thermometer.

Other practical temperature scales

Some laboratories did not adopt the ITS or IPTS and maintained temperature scales which, presumably, better approximate the thermodynamic scale. Some of these scales extended below the IPTS range as well. According to Keesom and Dammers[51] the Kamerlingh Onnes Laboratory (KOL) in Leiden, Netherlands, decided not to adopt the ITS. At KOL, temperatures were determined using platinum resistance thermometers which were calibrated by gas thermometry. In 1960 KOL established a hydrogen vapour pressure scale referred to as L–60. This scale is based on thermodynamic vapour pressure thermometry and magnetic thermometry, as reported by Durieux et al.[52]

Other temperature scales have been established at the Massachusetts Institute of Technology (MIT), at the Chemical Laboratory of the University of California (CAL) by Giauque et al[53,54] at the Ohio State University Cryogenic Laboratory (OSU) by Rubin et al,[55] and at the Physikalisch Technischen Reichansanstalt (PTR).

Intercomparison of thermodynamic and practical temperature scales

In this section the previously mentioned scales are intercompared. It would be desirable to compare the practical scales to the thermodynamic scale and, thus, indirectly to each other. However, the thermodynamic scale can only be realized by thermodynamic thermometry and thus, is constantly changing as measurement techniques improve. Because of this, it is desirable to make comparisons to recent, well documented, and highly precise scales. The scales chosen as a basis for a comparison having these characteristics and covering a wide range of temperatures are

1. 1958 He^4 scale, 1–5·2 K
2. NBS–55 scale, 10–90 K
3. IPTS–48 scale above 90 K

Comparisons from 1 to 5·2 K. The He^4 vapour pressure scales established previous to 1960 were compared by Brickwedde et al.[6] The difference tabulated by Brickwedde et al[6] are given in Table 5 and illustrated in Figure 4.

Plumb and Cataland's[50] comparisons of their acoustical thermometer measurements to the 1958 He^4 scale are also shown in Figure 4. The acoustical thermometer results in consistently higher temperatures varying linearly from 5 mK at 2 K to 12 mK at 5 K. Sydoriak et al[48] compared the 1962 He^3 scale to the 1958 He^4 scale; these data are not shown here since no systematic deviation is apparent between these two scales. The data of Sydoriak have a random scatter of $\pm 0{\cdot}6$ mK with a mean near zero.

Comparisons from 10 to 90 K. The three national scales (NBS, NPL, PRMI) and the PSU scale described previously, have been intercompared in a recent international cooperative effort. Six thermometers (one PRMI, one NBS, two NPL, and two PSU) calibrated on their own respective scales were compared both at NPL and at PRMI. These thermometers were inserted in a common copper block and the resistance of each was measured over the entire range from 10 to 90 K. The corresponding temperatures, as associated with each calibrated thermometer, indicated very precisely the differences between the four scales. The differences were reported by Orlova et al[56] and are illustrated in Figure 5. Other comparisons of some of these scales have been reported by Moessen, Aston, and Ascah,[14,57] Moessen,[58] Barber,[41,42,44] and Borovick-Romanov.[20] The data by Moessen et al were reported in tabular form and are included in Figure 5. The remaining data were reported in graphs too small for accurate retrieval and thus are not shown in Figure 5. I consider the measurements made at NPL and PRMI, as reported by Orlova et al[56] to be the more reliable, as indicated by the excellent agreement obtained by these two laboratories (see Figure 5). Plumb and Cataland's[26,49,50] comparisons of their acoustical thermometer measurements with the NBS–55 scale are also shown in Figure 5.

The data reported by Orlova et al[56] forms the basis for the IPTS–68 below 90 K. These data show that the oxygen and hydrogen boiling points were not realized identically by each of the laboratories, that is, the differences measured between these scales at these fixed points are not equal to the differences in the temperatures chosen for the oxygen and hydrogen boiling points by the various laboratories. To account for these descrepancies adjustments have been made to each of the scales in forming the basis for the IPTS–68 in this range. These adjusted data are not tabulated here, since the purpose of this paper is to compile information regarding the differences between scales as they actually exist in the calibration of thermometers.

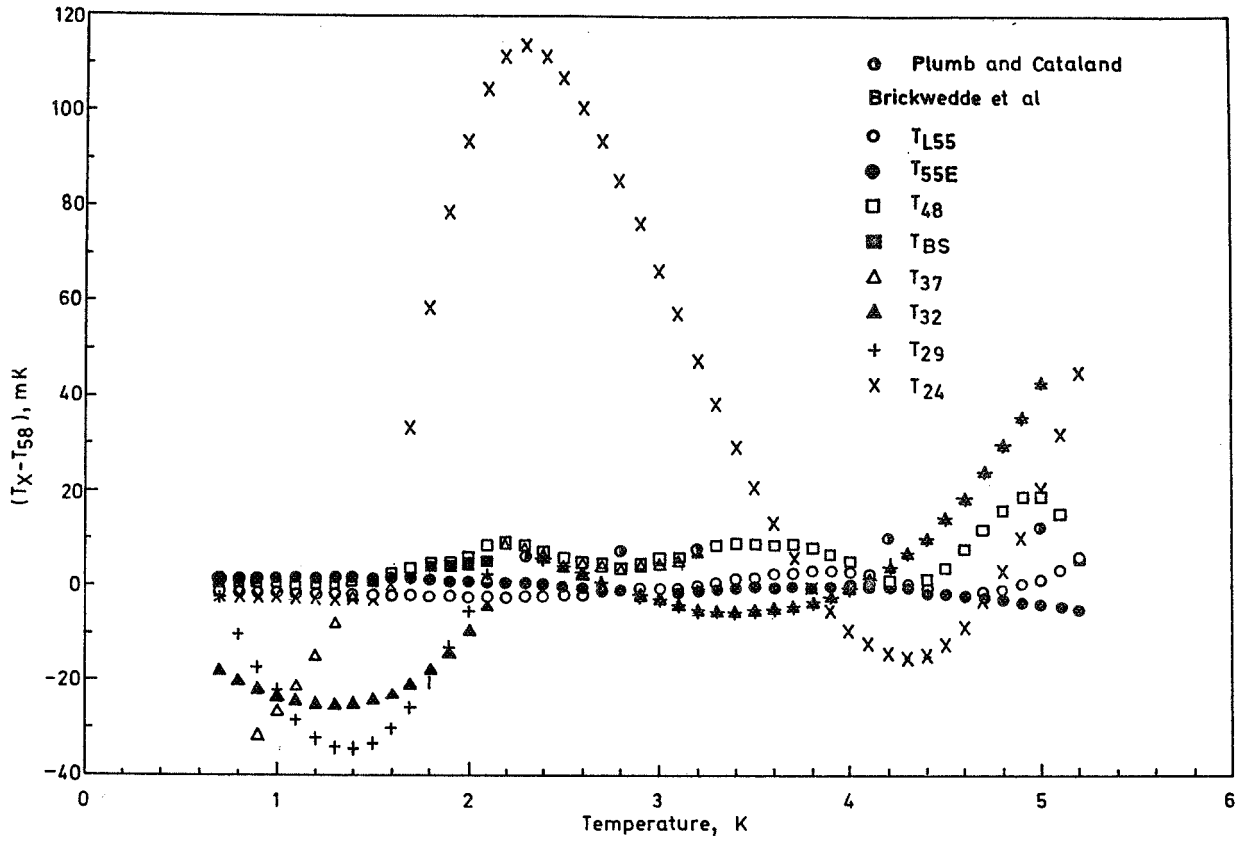

Figure 4. Deviations of earlier helium vapour pressure scales and the acoustical thermometer scale from the 1958 He^4 scale

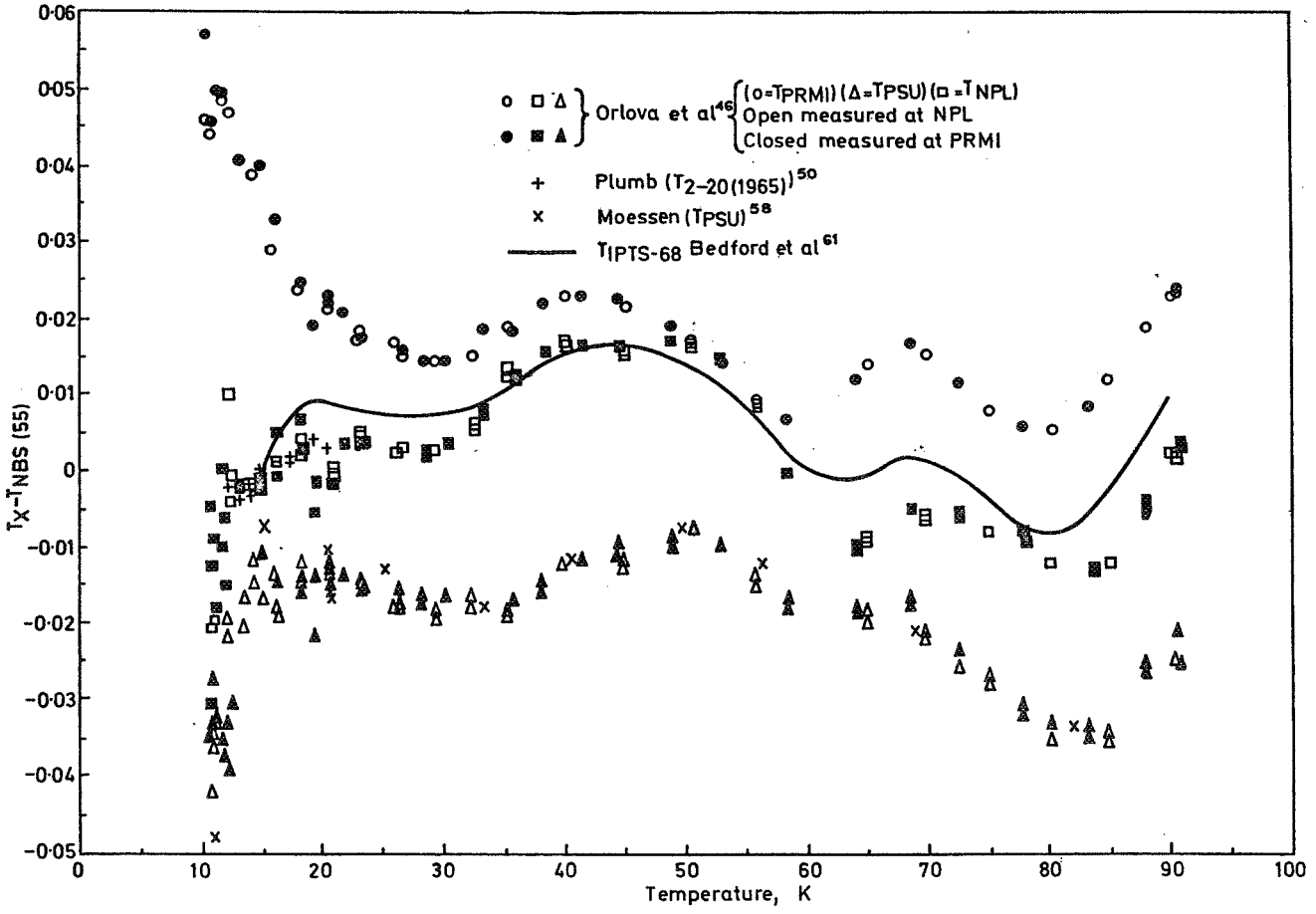

Figure 5. Comparison of low temperature scales (10 to 90 K), including PSU, NPL, PRIMI, NBS (2–20) 1965, and IPTS–68 with respect to the NBS (55) scale

TABLE 5. DEVIATIONS OF EARLIER He VAPOUR PRESSURE SCALES FROM THE 1958 He^4 SCALE, $T_n - T_{58}$ IN MILLIDEGREES[6]

T, K	T_{L55}	T_{55E}	T_{48}	T_{BS}	T_{37}	T_{32}	T_{29}	T_{24}
0·7	−1·1	+1·0	+0·4	+0·3	—	−18·4	−2·7	−2·1
0·8	−1·2	+1·1	+0·5	+0·3	—	−20·4	−10·3	−2·3
0·9	−1·3	+1·2	+0·6	+0·4	−31·9	−22·1	−17·3	−2·4
1·0	−1·5	+1·3	+0·3	+0·4	−26·8	−23·5	−23·5	−2·6
1·1	−1·6	+1·4	−0·2	+0·4	−21·5	−24·7	−28·5	−2·7
1·2	−1·7	+1·5	+0·4	+0·5	−15·2	−25·4	−32·1	−2·8
1·3	−1·8	+1·5	+0·3	+0·6	−8·1	−25·7	−34·1	−3·0
1·4	−1·9	+1·6	+1·0	+0·8	−2·3	−25·5	−34·5	−3·2
1·5	−2·0	+1·6	+1·3	+1·2	+1·1	−24·7	−33·2	−3·5
1·6	−2·1	+1·5	+2·9	+1·8	+2·6	−23·2	−30·3	+0·2
1·7	−2·1	+1·4	+3·8	+2·6	+3·2	−21·0	−25·9	+33·0
1·8	−2·2	+1·1	+3·9	+3·4	+3·5	−18·0	−20·2	+58·6
1·9	−2·2	+0·8	+5·1	+4·1	+4·2	−14·2	−13·3	+78·5
2·0	−2·2	+0·6	+6·0	+4·6	+5·6	−9·7	−5·6	+93·6
2·1	−2·2	+0·6	+8·4	+5·1	+7·1	−4·5	+2·5	+104·5
2·2	−2·2	+0·8	+9·4	—	+9·3	+8·2	+8·2	+111·2
2·3	−2·1	+0·5	+8·5	—	+8·2	+7·0	+7·0	+113·3
2·4	−2·0	0	+7·0	—	+6·9	+5·4	+5·4	+111·2
2·5	−1·9	−0·4	+5·6	—	+5·5	+3·8	+3·8	+106·8
2·6	−1·8	−0·7	+5·0	—	+4·4	+2·1	+2·1	+100·7
2·7	−1·5	−0·9	+4·5	—	+3·6	+0·6	+0·6	+93·4
2·8	−1·3	−1·1	+3·6	—	+3·4	−0·9	−0·9	+85·2
2·9	−0·9	−1·1	+4·7	—	+3·6	−2·2	−2·2	+76·3
3·0	−0·6	−1·1	+5·6	—	+4·2	−3·5	−3·5	+66·8
3·1	−0·2	−1·1	+6·0	—	+5·3	−4·5	−4·5	+57·2
3·2	+0·2	−0·9	+8·0	—	+6·7	−5·2	−5·2	+47·7
3·3	+0·7	−0·7	+8·6	—	+7·9	−5·7	−5·7	+38·4
3·4	+1·3	−0·5	+8·7	—	+8·7	−5·9	−5·9	+29·4
3·5	+1·9	−0·3	+8·8	—	+8·9	−5·8	−5·8	+21·0
3·6	+2·5	−0·1	+8·7	—	+9·0	−5·5	−5·5	+13·2
3·7	+2·9	0	+8·6	—	+8·6	−4·8	−4·8	+6·2
3·8	+3·2	0	+7·7	—	+7·5	−3·9	−3·9	−0·1
3·9	+3·2	−0·1	+6·6	—	+6·3	−2·6	−2·6	−5·4
4·0	+3·0	−0·2	+5·0	—	+4·7	−0·9	−0·9	−9·6
4·1	+2·4	−0·3	+2·6	—	+2·8	+1·2	+1·2	−12·7
4·2	+1·2	−0·6	+0·5	—	+1·1	+3·7	+3·7	−14·6
4·3	0	−1·0	−0·4	—	−0·5	+6·6	+6·6	−15·3
4·4	−0·9	−1·4	+0·9	—	—	+10·0	+10·0	−14·6
4·5	−1·5	−1·9	+3·8	—	—	+14·0	+14·0	−12·4
4·6	−1·7	−2·3	+7·7	—	—	+18·5	+18·5	−8·8
4·7	−1·5	−2·8	+11·9	—	—	+23·6	+23·6	−3·7
4·8	−0·9	−3·2	+15·8	—	—	+29·3	+29·3	+3·0
4·9	+0·2	−3·7	+18·6	—	—	+35·7	+35·7	+11·1
5·0	+1·7	−4·1	+19·0	—	—	+42·6	+42·6	+20·8
5·1	+3·7	−4·5	+15·8	—	—	—	—	+32·1
5·2	+6·3	−5·0	+6·5	—	—	—	—	+45·0

Explanatory notes concerning the deviations in Table 5 and Figure 4 are listed below.[6]

T_{24} Defined by the equation on p. 33 of *Leiden Comm. No. 147b* (Kamerlingh, Onnes, and Weber) and by last equation on p. 23 of *Leiden Comm. Suppl. No. 49* (Verschaffelt). These equations yield equal pressures at about 1·59 K. Therefore, values up through 1·5 K were derived from Verschaffelt's equation and those above from that of Kammerlingh, Onnes, and Weber.

T_{29} Defined by equation 6 on p. 36 of *Leiden Comm. No. 202c* (Keesom, Weber, and Schmidt). These equations give equal pressures at 2·1765 K. The authors state in the last paragraph of the communication, p. 37, that the first of the equation fits reasonably well the data of *Comm. No. 147b* up to 5 K. Therefore, deviations up to 5·0 K have been included in the table.

T_{32} Defined by the first of equation 6 on p. 36 of *Leiden Comm. No. 202c* (Keesom, Weber, and Schmidt) and by the equation on p. 8 of *Leiden Comm. No. 219a* (Keesom). T_{32} and T_{29} are thus identical above the lambda point. These two equations are discontinuous by about 0·008 K at 2·190 K and this fact was noted by Keesom (*Leiden Comm. Suppl. 71d*).

T_{37} Defined by T_{32} together with curves in Figure 1 of *Leiden Comm. No. 250c* (Schmidt and Keesom). Differences between T_{37} and T_{32} were determined directly from Figure 1 with sufficient precision to determine differences between T_{58} and T_{37} to 0·1 mK.

T_{BS} Defined by equation 9 and curve of Figure 4 on p. 1212 of *Trans. Faraday Soc.* 35 (Bleaney and Simon, 1939). Part, or all, of this scale is sometimes referred to as T_{39}. In order to obtain differences between this scale and T_{58}, pressures were calculated from the equation and curve with sufficient precision to yield differences to 0·1 mK.

T_{48} Defined by Tables I and II on pages T153 to T159 and by second equation on p. T152 of *Procès-Verbaux des séances du Comité International des Poids et Mesures* 23B, T151 (1952). Values given in the table were obtained in the following ways. First, values of the difference between this scale and the 1958 scale were calculated at every 0·01 interval between 0·95 and 4·25 K from data of Table II on pp. T158 and T159. The tabulated values from 1·0 through 4·2 K were then obtained by averaging the calculated differences between T − 0·05 and T + 0·05, with weights of $\frac{1}{2}$ assigned to the values at T − 0·05 to T + 0·05 and unit weights to all values at intermediate temperatures. For example, the tabulated value at 2·0 K is actually 1/20 of the sum of the differences at 1·96, 1·97, 1·98, 1·99, 2·00, 2·01, 2·02, 2·03, and 2·04 K. The tabulated values at 0·7, 0·8, and 0·9 K were obtained by calculating the pressure corresponding to each 0·01 interval between 0·65 and 0·95 K from the data on p. T153 of Table I and averaging the results in the manner just described. At 4·3 K and above, the tabulated values were obtained directly from the equation given on p. T152. Tables and equation have been published also by C. F. Squire, Low Temperature Physics, pp. 26, 229–33 (McGraw-Hill, New York, 1953).

T_{55E} Defined by the unnumbered equations on p. 188, 'Low temperature physics and chemistry' (Clement) in Proc. Fifth Intern. Conf. (Univ. of Wisconsin Press, Madison, Wis., 1958). Values of vapour pressure in millimetres mercury at 20 C were calculated by the computer at the US Naval Research Laboratory (the NAREC) and values in millimetres mercury at 0 C were calculated by the computer at Los Alamos Scientific Laboratory (the MANIAC). The MANIAC calculation was used for obtaining the values in the table.

T_{L55} Defined by Table 7 on p. 461, Progress in Low Temperature Physics (Van Dijk and Durieux),

(North-Holland Publishing Co., Amsterdam, 1957). Values below 0·9 K were obtained from the tables mentioned in section 23 of the reference. Table VII mentioned above is the same as Table V published in *Physica* 24, 1 (1958) and in *Leiden Comm. Suppl. 113c.*

Irregularities in the NBS–55 scale have been observed by Furukawa and Reilly[59] from heat capacity measurements and by Roder[60] from the *PVT* measurements.

Bedford[61] has derived differences between the IPTS–68 and the four national scales. These differences are considered precise to within about 0·1 mK, but may be inaccurate by as much as several millikelvins. The differences between IPTS–68 and NBS–55 scales as tabulated by Bedford[61] are shown in Figure 5.

Comparisons above 90 K. The ITS and IPTS–48 are in agreement below 600 C. The differences above 600 C, given by Corruccini,[31] are tabulated in Table 6 and illustrated in Figure 6. Differences between the international practical and the thermodynamic scales have been reported by many investigators. Some of these differences were noted even at the time of the adoption of the ITS. Differences were reported below 0 C in the platinum thermometer range by Heuse and Otto,[62] later by Keesom and Dammers,[63] and recently by Barber and Horsford,[64] Borovick-Romanov,[20] and Preston-Thomas and Kirby.[65] Irregularities in the IPTS–48 have been reported by Furukawa and Reilly[59] from heat capacity measurements, and by Roder[60] from *PVT* measurements. Differences between the international and the thermodynamic scales above 0 C have been reported by Blaisdell and Kaye,[66] Beattie,[67] Kirenkov et al,[16,68]

TABLE 6. DIFFERENCES BETWEEN ITS (1927) and IPTS (1948)

Platinum thermometer range

The temperature differences in this range are negligible. The changes were made primarily to make the scale more reproducible and definite. Note that the range on the 1948 scale is restricted to −182·97 to 630·5 C instead of −190 to 660 C.

Standard thermocouple range

t C(Int) 1948	Δt C(Int) 1948 −C(Int) 1927
630·5	0·00*
650	0·08*
700	0·24
750	0·35
800	0·42
850	0·43
900	0·40
950	0·32
1 000	0·20
1 050	0·05
1 063	0·00

* These values are uncertain since platinum thermometers are defined only up to 630·5 C on 1948 scale (see Corruccini[31]).

Radiation law range

It is difficult to determine exact differences of the ITS and IPTS in this range because of the variability of λ; the wavelength of the radiation on the 1927 scale is restricted to the visible spectrum and is not restricted at all on the 1948 scale. The following table contains the differences calculated at $\lambda_1 = 0{\cdot}4738 \times 10^{-4}$ cm and $\lambda_2 = 0{\cdot}65 \times 10^{-4}$ cm according to Corruccini.[31]

t C(Int) 1948	Δt C(Int) 1948 −C(Int) 1927	
	λ_1	λ_2
1 063	0	0
1 500	− 2	− 2
2 000	− 6	− 6
2 500	−12	−12
3 000	−19	−20
3 500	−28	−30
4 000	−39	−43

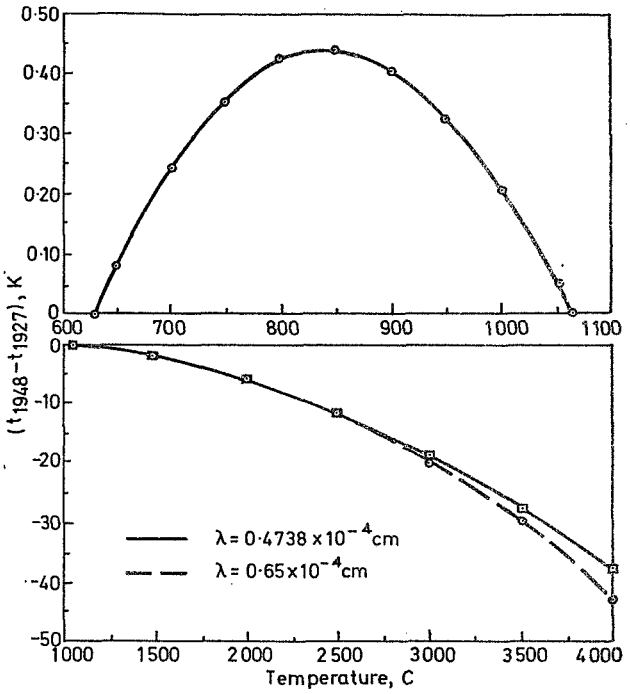

Figure 6. Temperature differences between the ITS and the IPTS–48

Oishi et al,[69] and Moser et al.[11,70,71] These data, illustrated in Figure 7, have been compiled and correlated by the CCT as the basis for the IPTS–68. Interim results of this effort have been presented by Terrien and Preston-Thomas.[72] Figure 7 contains the experimentally determined differences cited above and the differences between IPTS–48 and IPTS–68.[32] The latter differences are tabulated in Table 7.

Other comparisons. During the course of this investigation very little information has been obtained regarding intercomparison of the scales previously designated by MIT, KOL, CAL, PTR, and OSU. Durieux et al,[52] did indicate that the hydrogen vapour pressure scale established at KOL and designated as L–60 is lower than the NBS–55 scale by about 0·018 K from 15 to 23 K.

In the absence of direct comparisons of temperature scales, one may consider the temperature differences based upon the temperature differences of the fixed points of the scales. Hoge[73] compared the NBS–55, University of California, and Physikalisch Technischen Reichansanstalt scales by this method. These results are illustrated in Figure 8. The validity of this approach is dependent upon the deviations between two scales being linearly systematic between adjacent fixed points.

The method Hoge[73] used for the comparisons of the NBS, CAL, and PTR scales is given by (9) for any pair of adjacent fixed points. Equation 9 reduces to the same form as that used by Ziegler et al[21]

$$\Delta T = \frac{(T_1 - T_1') - (T_2 - T_2')}{T_1 - T_2}(T - T_2) + (T_2 - T_2') \quad \ldots (9)$$

where T_1' and T_2' are fixed point temperatures on the scale to be compared while T_1 and T_2 are the values of

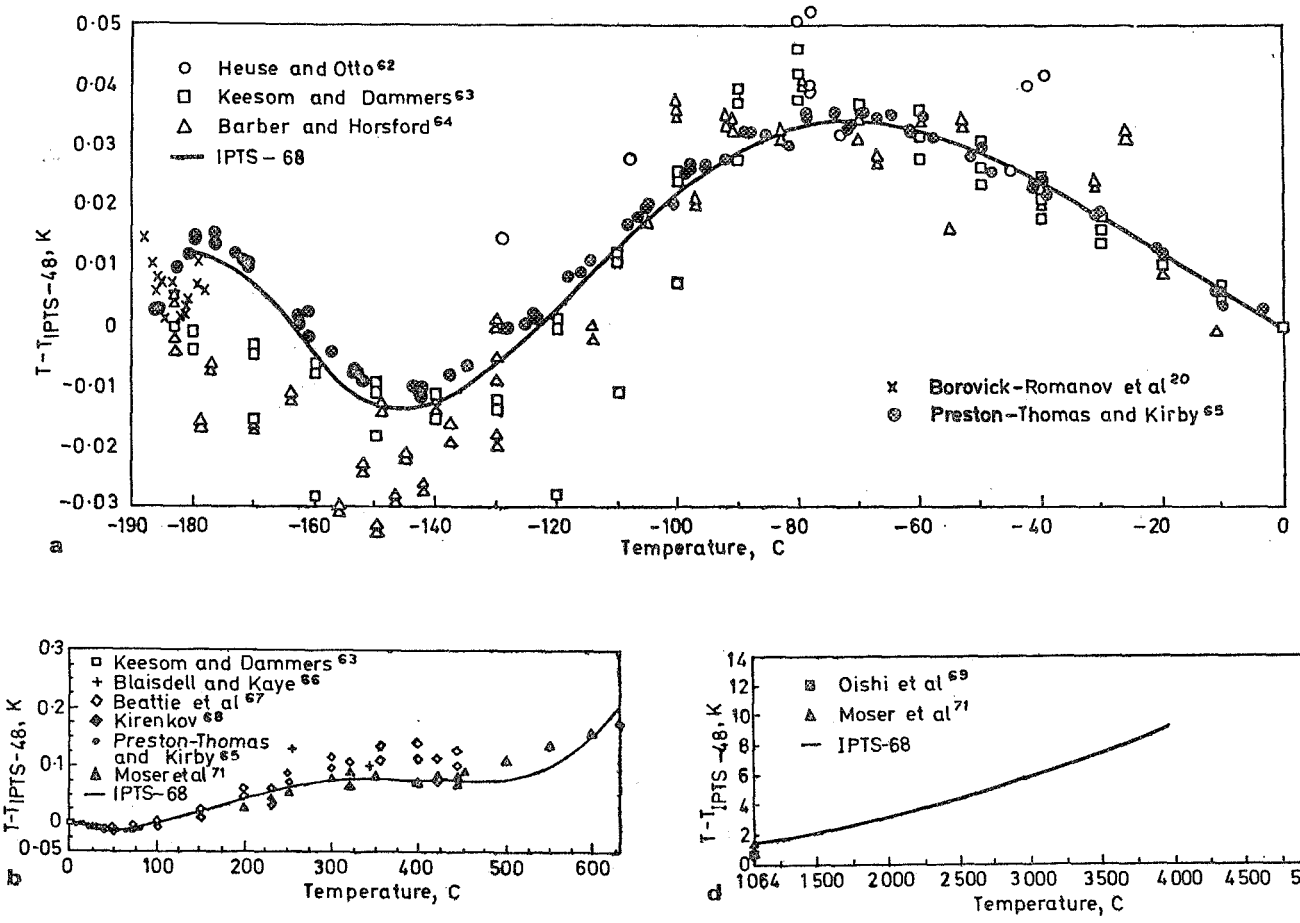

Figure 7. (a) Temperature differences of the thermodynamic and IPTS–68 scales from the IPTS–48 scale (−190 to 0 C) (b) Temperature differences of the thermodynamic and the IPTS–68 scales from the IPTS–48 scale (0 to 630 C) (c) Temperature differences of the thermodynamic and the IPTS–68 scales from the IPTS–48 scale (630 to 1 064 C) (d) Temperature differences of the thermodynamic and the IPTS–68 scales from the IPTS–48 scale (1 064 to 5 000 C)

TABLE 7. APPROXIMATE DIFFERENCES BETWEEN IPTS–68 AND IPTS–48,[32] VALUES TABULATED ARE ($T_{68} - T_{48}$) K

t_{68}, C	0	−10	−20	−30	−40	−50	−60	−70	−80	−90	−100
−100	0·022	0·013	0·003	−0·006	−0·013	−0·013	−0·005	0·007	0·012		
−0	0·000	0·006	0·012	0·018	0·024	0·029	0·032	0·034	0·033	0·029	0·022

t_{68}, C	0	10	20	30	40	50	60	70	80	90	100
0	0·000	−0·004	−0·007	−0·009	−0·010	−0·010	−0·010	−0·008	−0·006	−0·003	0·000
100	0·000	0·004	0·007	0·012	0·016	0·020	0·025	0·029	0·034	0·038	0·043
200	0·043	0·047	0·051	0·054	0·058	0·061	0·064	0·067	0·069	0·071	0·073
300	0·073	0·074	0·075	0·076	0·077	0·077	0·077	0·077	0·077	0·076	0·076
400	0·076	0·075	0·075	0·075	0·074	0·074	0·074	0·075	0·076	0·077	0·079
500	0·079	0·082	0·085	0·089	0·094	0·100	0·108	0·116	0·126	0·137	0·150
600	0·150	0·165	0·182	0·200	0·23	0·25	0·28	0·31	0·34	0·36	0·39
700	0·39	0·42	0·45	0·47	0·50	0·53	0·56	0·58	0·61	0·64	0·67
800	0·67	0·70	0·72	0·75	0·78	0·81	0·84	0·87	0·89	0·92	0·95
900	0·95	0·98	1·01	1·04	1·07	1·10	1·12	1·15	1·18	1·21	1·24
1 000	1·24	1·27	1·30	1·33	1·36	1·39	1·42	1·44			

t_{68}, C	0	100	200	300	400	500	600	700	800	900	1 000
1 000		1·5	1·7	1·8	2·0	2·2	2·4	2·6	2·8	3·0	3·2
2 000	3·2	3·5	3·7	4·0	4·2	4·5	4·8	5·0	5·3	5·6	5·9
3 000	5·9	6·2	6·5	6·9	7·2	7·5	7·9	8·2	8·6	9·0	9·3

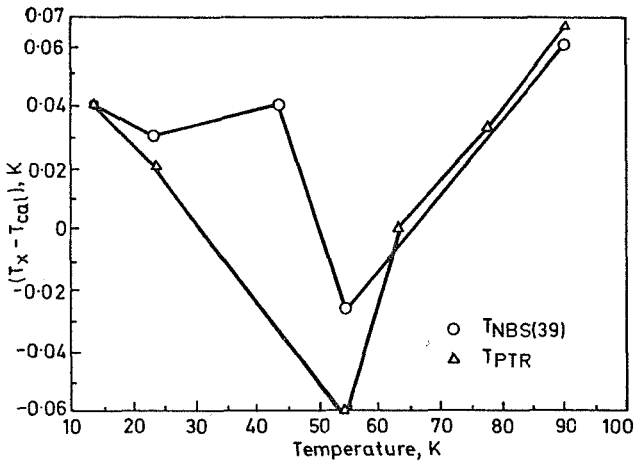

Figure 8. Temperature differences of the NBS (39) and PTR scales with respect to the CAL scale

these fixed points on the reference scale. ΔT is the indicated temperature difference of the two scales at a temperature T between T_1 and T_2.

A different method of reducing scales to common fixed points is reported by Orlova.[56] The temperature conversion according to Orlova's method results in

$$\Delta T = \frac{T}{T_1}(T_1 - T_1') + \left[T_2 - T_2' - \frac{T_2'}{T_1}(T_1 - T_1')\right]\frac{T - T_1}{T_2 - T_1} \quad \ldots (10)$$

The temperature differences, ΔT, obtained from (10) are for all practical purposes identical with those obtained from (9) when temperatures between the two fixed points are considered. Thus, both equations are equally recommended if the reader feels it is meaningful to make conversions on this basis.

Conversion of reported temperatures to the best estimate of the thermodynamic scale

As indicated previously, the purpose of this paper is to aid the user in converting temperatures to the thermodynamic temperature scale. Although hard and fast rules cannot be given to cover all of the situations which will be encountered, some general guidelines are as follows. The first step to be taken is to make a very thorough and careful reading of the document to determine the temperature scale used by the author, the fixed points on which the scale is based, and the accuracy to which the scale was reproduced by the measuring instrument. The reader should always be alert for the possibility that the author has already made corrections to the measured temperatures. In a few instances none of this information will be reported, in which case, the reader is forced to accept the data as reported. More often, at least some information will be reported. Generally, the next step is to correct for variations in fixed points. Equation 9 or 10 is recommended for the fixed point corrections. These equations, although usually not exactly applicable, are recommended here because the deviations from more exact relations are small and because the information required to make more exact corrections is hardly ever presented. If the resulting scale is one of those described in this report, the last step is to obtain the conversion adjustment from the appropriate graph or table.

If the temperature scale used by a given investigator is not one compiled in this paper, one may resort to other techniques. For example, indirect comparisons of temperature scales can be made by comparing values for a given property as reported by several investigators on different temperature scales. This involves the assumption that the differences in reported property values are caused primarily by the difference in temperature scales. This assumption is best satisfied for accurate measurements of those properties which are readily reproduced and well defined. The vapour pressure of a material is such a property and so may be used with reasonable confidence to indicate temperature differences between two scales. Since some of the older pre-ITS scales are not well documented in a comparative sense, this may be the best procedure to convert temperatures reported on these scales to the thermodynamic scale. It is anticipated that, as time permits, such comparisons will be carried out and documented at this laboratory and will serve as a valuable supplement to this work.

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